



# Local electric field investigation of $\text{Si}_2\text{N}_2\text{O}$ and its electronic structure, elastic and optical properties

Ting Zhang<sup>a</sup>, Mengqiang Wu<sup>a,\*</sup>, Shuren Zhang<sup>a</sup>, Song Chen<sup>a</sup>, Ming He<sup>a</sup>, Jinming Wang<sup>b</sup>, Dahai Zhang<sup>b</sup>, Fengmei He<sup>b</sup>, Zhongping Li<sup>b,\*\*</sup>

<sup>a</sup> State Key Laboratory of Electronic Thin Films and Integrated Devices, University of Electronic Science and Technology of China, PR China

<sup>b</sup> Aerospace Research Institutes of Materials and Processing Technology, PR China

## ARTICLE INFO

### Article history:

Received 25 June 2010

Received in revised form

30 September 2010

Accepted 7 October 2010

Available online 15 October 2010

### Keywords:

First principles

$\text{Si}_2\text{N}_2\text{O}$

Local electric field

Dielectric constant

Additivity rule

## ABSTRACT

This paper demonstrates that the Clausius–Mosotti equation cannot be used for  $\text{Si}_2\text{N}_2\text{O}$  since the Lorenz approximation is invalid therein. Therefore a modified definition for the Lorenz electric field is suggested which can be derived from the optical dielectric constant calculated using the plane-wave pseudopotential method. In addition, other parameters of  $\text{Si}_2\text{N}_2\text{O}$  such as the energy band gap, density of states, elastic and optical properties are also given in this paper. Based on the new expression for local electric field, the modified Clausius–Mosotti equation is suggested, and then the dielectric constant of  $\text{Si}_2\text{N}_2\text{O}$  is discussed by using the additivity rule. It is found that the result of the phenomenological analysis of the dielectric constant is basically consistent with the experimental data and the first principles results, which explain the experimental observation that the dielectric constant of  $\text{Si}_2\text{N}_2\text{O}$  is enhanced with the increase of Li content. Also, the modified Clausius–Mosotti equation and the additivity rule are suggested to be used in predicting the dielectric behaviors of the new and complex compounds.

© 2010 Elsevier B.V. All rights reserved.

## 1. Introduction

Silicon oxynitride ( $\text{Si}_2\text{N}_2\text{O}$ ) is a unique compound in the  $\text{SiO}_2$ – $\text{Si}_3\text{N}_4$  quasi-binary [1]. Like  $\text{Si}_3\text{N}_4$ ,  $\text{Si}_2\text{N}_2\text{O}$ , it is an engineering material which can keep excellent oxidation resistance in air of up to 1600 °C and high flexural strength up to 1400 °C without degradation [2–5]. It is also an important link between two major classes of materials of great technological importance,  $\text{Si}_3\text{N}_4$  and  $\text{SiO}_2$ . Many theoretical studies have concerned on the electronic structure of  $\text{Si}_2\text{N}_2\text{O}$  due to the fact that the electronic structure calculations have become more important in the fields of physics and chemistry. On the basis of the first principles of orthogonalized-linear-combination-of-atomic-orbitals (OLCAO) method, Ching et al. [6] has predicted that the indirect band gap of  $\text{Si}_2\text{N}_2\text{O}$  is 5.97 eV whereas its valence-band DOS can be approximately regarded as a superposition of  $\alpha$ - $\text{SiO}_2$  and  $\beta$ - $\text{Si}_3\text{N}_4$  crystals with an exception at the top of the valence-band edge, which is derived from the oxygen lone-pair orbitals. Because of the feasi-

bility of using  $\text{SiO}_x\text{N}_y$  glass as a gradient index optical material [7], understanding on the intrinsic optical properties of the  $\text{Si}_2\text{N}_2\text{O}$  solids is certainly a matter of great importance. Xu and Ching [8] have calculated the optical dielectric function of  $\text{Si}_2\text{N}_2\text{O}$  by the means of self-consistent LDA calculations of the first principles. But limited reports have concerned on other optical properties such as reflectivity, loss function and conductivity by using the calculation method of the first principles.

Recently, a new synthesizing method to prepare  $\text{Si}_2\text{N}_2\text{O}$  with  $\text{Li}_2\text{O}$  additive gives good dielectric properties ( $\epsilon' = 6.17$ ,  $\tan \delta = 0.0008$ ) [9]. Thus, the good dielectric properties of  $\text{Si}_2\text{N}_2\text{O}$  mean that it can be used for many potential applications such as high temperature electric insulator, nuclear-reactor moderator or reflector, and materials for solid electrolytes. However, little work has been carried out on the theoretical prediction of the dielectric constant of  $\text{Si}_2\text{N}_2\text{O}$  based on the investigation of its local electric field.

As is well known, the Clausius–Mosotti formula gives a remarkably good approximation to the effective dielectric constant of mixtures and materials containing several phases [10,11]. This equation can be given as [12],

$$\frac{\epsilon - 1}{\epsilon - 1 + (4\pi/L)} = \frac{L}{V_m} \alpha_m \quad (1)$$

\* Corresponding author at: State Key Laboratory of Electronic Thin Films and Integrated Devices, University of Electronic Science and Technology of China, PR China.

\*\* Corresponding author at: Aerospace Research Institutes of Materials and Processing Technology, PR China.

E-mail addresses: [mwu@uestc.edu.cn](mailto:mwu@uestc.edu.cn) (M. Wu), [office@arimt.com](mailto:office@arimt.com) (Z. Li).



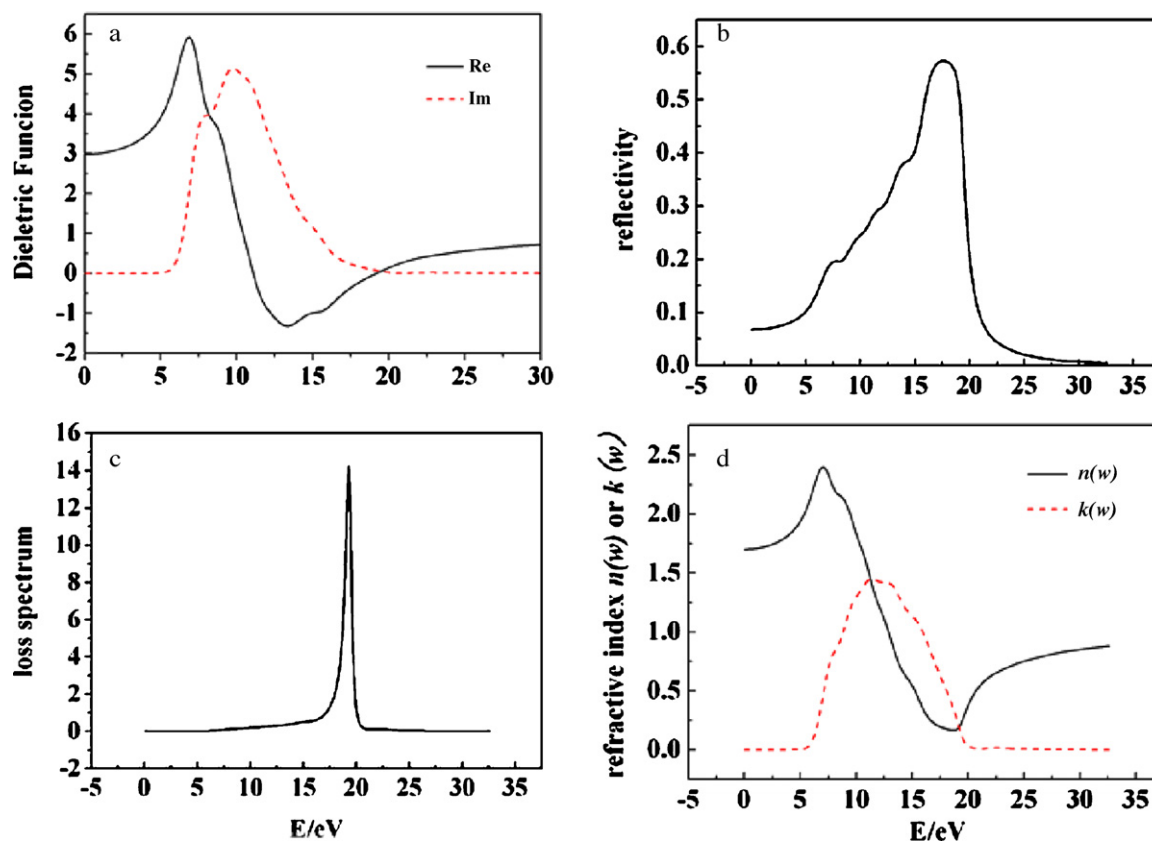


Fig. 2. The calculated optical properties for Si<sub>2</sub>N<sub>2</sub>O.

Si<sub>2</sub>N<sub>2</sub>O,  $A_B = 0.43\%$  and  $A_G = 5.04\%$  with our DFT calculation. These results indicate that Si<sub>2</sub>N<sub>2</sub>O is slightly anisotropic in compressibility while it is greatly anisotropic in shear modulus. Moreover, the ratio  $B/G$  can be used to estimate the ductility of the material [28–30], and from the bulk and shear modulus calculated in this work for Si<sub>2</sub>N<sub>2</sub>O, one obtains  $B/G = 1.65$ . This  $B/G$  ratio suggests that the Si<sub>2</sub>N<sub>2</sub>O crystal has an increasing toughness of more than 98% compared with diamond.

### 3.2. Optical properties

Fig. 2 shows the calculated optical properties are at the equilibrium lattice constant in an energy ranging from 0 to 35 eV. To account for the structures observed in the optical spectra, it is customary to consider transitions from the occupied to unoccupied bands in the electronic energy band structure especially at high symmetry points in the Brillouin zone. The calculated real and imaginary parts of dielectric function for Si<sub>2</sub>N<sub>2</sub>O from the Kramers–Kronig analysis are shown in Fig. 2(a). The zero frequency limit of  $\epsilon_1(\omega)$  is the electronic part of the static optical dielectric constant  $\epsilon_1(0)$ , which is sometimes denoted as  $\epsilon_\infty$ . Obviously, it is noted from the Fig. 2(a) that the calculated static optical dielectric constant  $\epsilon_\infty$  of Si<sub>2</sub>N<sub>2</sub>O is 2.98. This value is in excellent consistent with the results of other methods [6,8]. Also, it is seen that the real and imaginary parts of dielectric function have one peak at about 6 eV and 5 eV, respectively. This energy peak is attributed to the transition of electron excitation from the VB to the CB in the electronic energy band structure. The calculated reflectivity spectrum, i.e.,  $R(\omega)$  and loss spectrum, i.e.,  $L(\omega)$ , are shown in Fig. 2(b) and (c). The electron energy loss function  $L(\omega)$  is an important factor describing the energy loss of a fast electron traversing in a material. Prominent peaks in  $L(\omega)$  spectra represent the characteristics associated with the plasma oscillations and the corresponding

frequencies are the so-called bulk plasma frequency  $\omega(p)$ , which occurs where  $\epsilon_2 < 1$  and  $\epsilon_1$  reaches zero point [31]. Obviously, one may note from the Fig. 2(b) and (c) that the peak of  $L(\omega)$  is at about 17.58 eV, which corresponds to an abrupt reduction of  $R(\omega)$ . The refractive index provides useful information about the optical properties of the material. The extinction coefficient directly describes the attenuation of electromagnetic waves within the material and is also known as a damping constant or attenuation coefficient. The refractive index and the extinction coefficient are given in Fig. 2(d). The static refractive index  $n(0)$  is found to have a value of 1.70. This value increases with the energy increasing in the transparency region and reaches its peak in the ultraviolet at about 2.40 eV. It is decreased to the minimum at 0.17 eV. The origin of the structures in the imaginary part of the dielectric function also explains the structures in the refractive index. The local maxima of the extinction coefficient  $k(\omega)$  corresponds to the zero of  $\epsilon_1(\omega)$ . Our calculated optical constants of the dielectric function, absorption spectrum, refractive index, extinction coefficient, reflectivity, and energy loss coefficient are in good agreement with the results of other calculation methods [8] and can be used as reference data for analyzing the optical properties of more complicated SiO<sub>x</sub>N<sub>y</sub> compounds.

### 3.3. Local electric field

It is assumed that the actual electric field  $E$  is seen by a dipole, i.e., its local electric field,  $E_{\text{local}}$ , is given by the following:

$$E_{\text{local}} = E_a + E_{\text{LOR}} + E_{\text{near}} \quad (3)$$

where,  $E_a$  stands for the external electric field,  $E_{\text{LOR}}$  for the contribution of the atoms or ions outside the sphere (called as Lorenz electric field), and  $E_{\text{near}}$  for the contribution of the sources inside the sphere while excluding the central dipole (on the principle that the electric field of the central dipole does not act on itself).

**Table 2**  
The value of  $\varepsilon_\infty$  of different  $\gamma$ .

$\gamma$	0	0.2	0.5	0.8	1.0
$\varepsilon_\infty$	3.04	3.38	4.09	6.82	7.35

There are two assumptions introduced to the Lorenz approximation. Firstly, the spatial dimension of a molecule is assumed to be small enough that the local electric field can be taken to be consistent with the whole molecule. In this premise, the polar particle is treated as a point-dipole. However, if the distance of the ions is short, the point-dipole approximation is invalid therein, and the interaction between the particles cannot be ignored. Secondly, it is assumed that the near electric field ( $E_{\text{near}}$ ) can be approximately considered as the sum of the contributions from the point dipoles at the lattice points inside the auxiliary sphere. If the crystal has a cubic symmetry, the electric field of the sources inside the sphere would be zero, i.e.,  $E_{\text{near}} = 0$ . It is known that  $\text{Si}_2\text{N}_2\text{O}$  has an asymmetric structure which belongs to the space group  $C_{2v}^{12}$ . Hence, the contributions of the sources inside the sphere may not be ignored. Accordingly, it is necessary to consider the contributions of the overlap electron clouds to the local electric field, simple to say, a modified definition should be introduced to the local electric field of  $\text{Si}_2\text{N}_2\text{O}$ . Eq. (1) is accordingly replaced by,

$$\frac{\varepsilon - 1}{\gamma(\varepsilon - 1) + (4\pi/L)} = \frac{L}{\varepsilon_0 V_m} \alpha_m \quad (4)$$

where  $\gamma$  stands for the modified definition for the local electric field,  $0 \leq \gamma \leq 1$ .

Also, the optical dielectric constant  $\varepsilon_\infty$  is given by,

$$\frac{\varepsilon_\infty - 1}{\gamma(\varepsilon_\infty - 1) + (4\pi/L)} = \frac{L}{\varepsilon_0 V_m} \alpha_e \quad (5)$$

where  $\alpha_e$  represents electronic polarizability of  $\text{Si}_2\text{N}_2\text{O}$ . Correspondingly, one may note from Eq. (5) that the value of  $\gamma$  can be determined by  $\varepsilon_\infty$  and  $\alpha_e$ . The value of electronic polarizability  $\alpha_e$  of  $\text{Si}_2\text{N}_2\text{O}$  is known as,

$$\begin{aligned} \alpha_e(\text{Si}_2\text{N}_2\text{O}) &= 2\alpha_e(\text{Si}^{4+}) + 2\alpha_e(\text{N}^{3-}) + \alpha_e(\text{O}^{2-}) \\ &= 10.45 \times 10^{-40} \text{ F} \cdot \text{m}^2 \end{aligned} \quad (6)$$

Therefore, we can obtain the value of  $\varepsilon_\infty$  by combining Eqs. (5) and (6) with different value of  $\gamma$ , seen in Table 2. And  $\varepsilon_\infty$  can be obtained by a plane-wave pseudopotential method in the Section 3.2, which is equal to 2.98. Therefore, it is deduced that an appropriate factor of  $\gamma$  is approximately equal to zero from Table 2. If  $\gamma = 0$ , the expression (5) has the following form,

$$\frac{\varepsilon - 1}{4\pi} = \frac{\alpha_m}{\varepsilon_0 V_m} \quad (7)$$

It means the polarized field of  $\text{Si}_2\text{N}_2\text{O}$  generated by the polarized ions is equal to the external applied electric field for the reason that the overlapping electronic clouds of  $\text{Si}_2\text{N}_2\text{O}$  are so great that they would make the Lorenz electric field, i.e.,  $E_{\text{LOR}} = P/3\varepsilon_0$  be counteracted by the electric field generated by the polarized ions ( $E_{\text{near}}$ ).

### 3.4. Dielectric constant and the additivity rule

The concept of additivity of molar dielectric polarizabilities according to,

$$\alpha_m(\text{complex mixture}) = \sum \alpha_m(\text{oxide components}) \quad (8)$$

has been discussed by Lasaga and Cygan [32]. In the above expression, the molecular polarizabilities ( $\alpha_m$ ) of the complex crystal can be expressed as the summation of the polarizabilities of the constituent crystal. Thus the dielectric polarizabilities and the

dielectric constants of new materials or compounds whose dielectric constants have not been measured are potentially predictable by the polarizability additivity rules. On the basis of the dielectric constants of 129 oxides and 25 fluorides, Shannon [33] has derived a set of 61 ion dielectric polarizabilities from a least squares refinement procedure by using the ion additivity rule and calculated dielectric constants for about a hundred compounds in conjunction with the Clausius–Mosotti equation. Excellent agreement between the calculated polarizability values and those measured data is shown for many ternary systems such as borates, aluminates, gallates, silicates, germinates, phosphates, and vanadates, etc. So, the Clausius–Mosotti equation, and the additivity rule can be used in combination as a powerful tool for predicting the dielectric constant of any hypothetical materials. However, these discussions are based on the Clausius–Mosotti equation and here it should be necessary to ask if this additivity rule can be used for  $\text{Si}_2\text{N}_2\text{O}$ . According to the above concept of the additivity rules of molecular polarizabilities, the molecular polarizabilities  $\text{Si}_2\text{N}_2\text{O}$  can be obtained from a sum of the molecular polarizabilities of the  $\text{Si}_3\text{N}_4$  and  $\text{SiO}_2$ ,

$$\alpha_m(\text{Si}_2\text{N}_2\text{O}) = \frac{1}{2} \alpha_m(\text{SiO}_2) + \frac{1}{2} \alpha_m(\text{Si}_3\text{N}_4) \quad (9)$$

The static polarizability values for  $\text{SiO}_2$  which is  $4.28 \times 10^{-40} \text{ Fm}^2$  [33] utilizing the Clausius–Mosotti equation with the dielectric data, and the static polarizability values for  $\text{Si}_3\text{N}_4$  is found to be  $42.89 \times 10^{-40} \text{ Fm}^2$  according to the measured dielectric constant [34]. Correspondingly, the calculated dielectric constant is 5.63 according to Eqs. (7)–(9), which may not be consistent with the measured value 6.17 by Tong et al. [9]. However, it is found that in their experiment, some  $\text{Li}_2\text{O}$  has been used for additive to prepare  $\text{Si}_2\text{N}_2\text{O}$ , and it is observed that the dielectric constant of  $\text{Si}_2\text{N}_2\text{O}$  is enhanced with the increase of residual Li content [9]. Therefore, when a comparison is made between the predicted value and the measured value from Tong et al, the effect of Li on the dielectric constant of  $\text{Si}_2\text{N}_2\text{O}$  should be considered. Since Li easily introduces  $\text{Si}_2\text{N}_2\text{O}$  to form the point defects due to its small size, the impact of Li on the polarization of  $\text{Si}_2\text{N}_2\text{O}$  may result in the orientation polarization. The total polarizability  $\alpha_m$  and the dielectric constant of  $\text{Si}_2\text{N}_2\text{O}$  with  $\text{Li}_2\text{O}$  additive  $\varepsilon_{\text{dope}}$  thus should be expressed as follows:

$$\alpha_m = \alpha_m(\text{pure}) + \alpha_d \quad (10)$$

$$\frac{\varepsilon_{\text{dope}} - 1}{4\pi} = \frac{\alpha_m(\text{pure})}{\varepsilon_0 V_m} + \frac{N_d \alpha_d}{4\pi \varepsilon_0} \quad (11)$$

Where the orientation polarizability  $\alpha_d$  and the number  $N_d$  of impurity ions or defects can be given as,

$$\alpha_d = \sum \frac{(q\delta)^2}{12KT} \quad (12)$$

$$N_d = \sum \frac{p_{im} \rho_0 N_A}{m_{im}} \quad (13)$$

where  $q$  is the charge of impurity ions or defects.  $\delta$  is the typical jumping distance, which has the same order as the lattice constants,  $m_{im}$  is the molar mass, and  $p_{im}$  is the concentration and the value is from the measurement [9]. Using Eqs. (11)–(13), it is found that the polarization contribution caused by Li is 0.53. Thus the calculated value of  $\text{Si}_2\text{N}_2\text{O}$  with  $\text{Li}_2\text{O}$  additive is 6.16, which agrees with the measured value 6.17 within the approximation of 0.16% uncertainty. Also, it is seen from Eqs. (11)–(13) that more concentration of Li would lead to higher dielectric constant due to the fact that more polarized ions are generated. The conclusions is key to explain the experimental observation by Tong et al. [9], who has found that the polycrystalline dielectric constant of  $\text{Si}_2\text{N}_2\text{O}$  is enhanced with the increase of residual Li content.

Recently, the effects of impurity Li atom on crystal structure, electronic and dielectric properties of  $\text{Si}_2\text{N}_2\text{O}$  are calculated by Liu et al. [35] using a Plane-wave pseudopotential total energy method and it is found that the dielectric constant of Li-doped  $\text{Si}_2\text{N}_2\text{O}$  is increased by 0.60 compared to that of pure one, which can be attributed to a reduction of band gap. Obviously, our phenomenological analysis of dielectric constant is very well consistent with their results, which reveals that our calculation method is reliable to predict the dielectric constant of  $\text{Si}_2\text{N}_2\text{O}$ . Also, the consistence demonstrates that the modified Clausius–Mosotti equation and the additivity rule are suggested to be used in predicting the dielectric constant of  $\text{Si}_2\text{N}_2\text{O}$ , which means that the dielectric behaviors of the new and complex compounds with asymmetric structures can be obtained from those of the simple materials by adopting this method.

#### 4. Conclusions

In summary, it is found that the Lorenz approximation cannot be used in  $\text{Si}_2\text{N}_2\text{O}$ , which shows that the interaction between the polar particles cannot be ignored. Therefore, the local electric field of the  $\text{Si}_2\text{N}_2\text{O}$  is analyzed by introducing a modified definition for the Lorenz electric field since it may dominate the polar particles motion. The modified definition parameter of local electric field is found to be equal to zero by using its optical dielectric constant, which shows that the overlap electron clouds of  $\text{Si}_2\text{N}_2\text{O}$  would make the Lorenz electric field, i.e.,  $E_{\text{LOR}} = P/3\epsilon_0$  counteract by the electric field of the polar ions ( $E_{\text{SPH}}$ ). And based on the determination of the modified definition of local electric field of  $\text{Si}_2\text{N}_2\text{O}$  and the additivity rule, its dielectric constant is studied. By making the comparison of the calculated results with that of other researchers, it is found that the modified Clausius–Mosotti equation and the additivity rule can be used to obtain the dielectric behaviors of the new and complex compounds.

#### Acknowledgement

We gratefully acknowledge the support of the Scientific Research Fund of Sichuan Provincial Education Department (No.09ZC029), Chengdu Medical Foundation (No.CYZ08-016).

#### References

- [1] L. Kaufman, *Calphad* 3 (1979) 275–291.
- [2] M. Ohashi, S. Kanzaki, H. Tabata, *J. Mater. Sci. Lett.* 7 (1988) 339–340.
- [3] M. Ohashi, S. Kanzaki, H. Tabata, *J. Am. Ceram. Soc.* 74 (1991) 109–114.
- [4] M. Ohashi, S. Kanzaki, H. Tabata, *J. Mater. Sci.* 26 (1991) 2608–2614.
- [5] Z.K. Huang, P. Greil, G. Petzow, *Ceram. Int.* 10 (1984) 14–17.
- [6] W.Y. Ching, S.Y. Ren, *Phys. Rev. B* 24 (1981) 5788–5795.
- [7] T. Baak, *Appl. Opt.* 21 (1982) 1069–1072.
- [8] W.Y. Ching, *J. Am. Ceram. Soc.* 87 (2004) 1996–2013.
- [9] Q.F. Tong, Y.C. Zhou, J.Y. Wang, et al., *J. Eur. Ceram. Soc.* 27 (2007) 4767–4772.
- [10] R.R. Reddy, K. Rama Gopal, K. Narasimhulu, *J. Alloys Compd.* 473 (2009) 28–35.
- [11] G.B. Sakr, I.S. Yahia, *J. Alloys Compd.* 503 (2010) 213–219.
- [12] C. Kittel, *Introduction to Solid State Physics*, Wiley, New York, 1968.
- [13] J.M.F. Gunn, M. Qrtuno, *J. Phys. C: Solid Phys.* 13 (1980) 1669–1678.
- [14] A.P. Vinogradov, *Phys. A* 241 (1997) 216–222.
- [15] L. Banyai, P. Gartner, *Phys. Rev. B* 29 (1984) 728–734.
- [16] Y. Yang, H. Lu, C. Yu, J.M. Chen, *J. Alloys Compd.* 485 (2009) 542–547.
- [17] K.L. Zhao, D. Chen, D.X. Li, *J. Alloys Compd.* 485 (2009) 598–603.
- [18] F. Peng, D. Chen, X.D. Yang, *J. Alloys Compd.* 489 (2010) 140–145.
- [19] M. Othman, E. Kasap, N. Korozlu, *J. Alloys Compd.* 496 (2010) 226–233.
- [20] K. Haddadi, A. Bouhemadou, L. Louail, *J. Alloys Compd.* 504 (2010) 296–302.
- [21] X. Huang, J. Lv, Z.S. Li, Z.G. Zou, *J. Alloys Compd.* 507 (2010) 341–344.
- [22] I. Idrestedt, C. Brosset, *Acta Chem. Scand.* 18 (1964) 1879–1886.
- [23] J.C. Labbe, M. Billy, C. R. Acad. Sci. Ser. C 277 (1973) 1137–1140.
- [24] B. Liu, J.Y. Wang, F.Z. Li, et al., *J. Phys. Chem. Solids* 70 (2009) 982–988.
- [25] H.Z. Yao, L.Z. Ouyang, W.Y. Ching, *J. Am. Ceram. Soc.* 90 (2007) 3194–3204.
- [26] P. Boch, J.C. Glandus, *J. Mater. Sci.* 14 (1979) 379–385.
- [27] K. Lau, A.K. McCurdy, *Phys. Rev. B* 58 (1998) 8980–8984.
- [28] P. Ravindran, L. Fast, P.A. Korzhavyi, et al., *J. Appl. Phys.* 84 (1998) 4891–4904.
- [29] S.F. Pugh, *Philos. Mag.* 45 (1954) 823–843.
- [30] C.A. Perotoni, J.A.H. da Jornada, *Phys. Rev. B* 65 (2002) 224208(1)–224208(6).
- [31] D. Almeida, J.S. Ahuja, *Phys. Rev. B* 73 (2006) 165102(1)–165102(2).
- [32] A.C. Lasaga, R.T. Cygan, *Am. Miner.* 67 (1982) 328–334.
- [33] R.D. Shannon, *J. Appl. Phys.* 73 (1993) 348–366.
- [34] J.D. Walton Jr., *Bull Am Ceram Soc.* 53 (1974) 255–258.
- [35] B. Liu, J.Y. Wang, F.Z. Li, et al., *J. Mater. Sci.* 44 (2009) 6416–6422.